Calculus Warm Up #1-3

1. Find the explicit rule for the geometric sequence with 4th term 3010 and 7th term 3,010,000.

2. Graph the sequence in Parametric Mode. Does it appear to converge or diverge?

$$a_n = \frac{\cos n}{n}$$

Does the sequence converge? If so, find the limit.

$$a_n = \frac{\cos n}{n} \qquad \lim_{n\to\infty}\left(\frac{\cos n}{n}\right)$$

But we know: $-1 \le \cos n \le 1$

$\div$ by n $\longrightarrow$ $-\frac{1}{n} \le \frac{\cos n}{n} \le \frac{1}{n}$

take the limit $\rightarrow$ $\lim_{n\to\infty}\left(-\frac{1}{n}\right) \le \lim_{n\to\infty}\left(\frac{\cos n}{n}\right) \le \lim_{n\to\infty}\left(\frac{1}{n}\right)$

Evaluate the ends $\longrightarrow$ $0 \le \lim_{n\to\infty}\left(\frac{\cos n}{n}\right) \le 0$

$\therefore \lim_{n\to\infty}\left(\frac{\cos n}{n}\right) = 0$

Squeeze Theorem

(also called the Sandwich Theorem)

If $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$

And there exist an integer, N, such that

$a_n \leq c_n \leq b_n$ for all $n > N$

Then $\lim_{n\to\infty} c_n = L$

Comparing growth rates

$$\lim_{n\to\infty} \frac{2^n}{n!}$$

n	2^n	$n!$
2	4	$1\cdot 2 = 2$
3	8	$1\cdot 2\cdot 3 = 6$
4	16	$6\cdot 4 = 24$ ← $n!$ overtakes 2^n
5	32	$24\cdot 5 = 120$

As $n \to \infty$ $n!$ has a much bigger growth rate than 2^n

So as $n \to \infty$

$$\frac{2^n}{n!} \leftarrow \frac{\text{a number}}{\text{a much bigger \#}} \rightarrow \frac{c}{\cancel{\infty}} \nearrow 0$$

$$\lim_{n\to\infty} \frac{n!}{8^n}$$

It will take longer for $n!$ to overtake 8^n, but as $n \to \infty$

$$\frac{n!}{8^n} \to \frac{\infty}{\text{a much smaller \#}} \to \infty$$

HW Questions: p. 570

In Exercises 1–8, write out the first five terms of the sequence with the given nth term.

1. $a_n = 2^n$

3. $a_n = \left(-\frac{1}{2}\right)^n$

5. $a_n = \frac{3^n}{n!}$

7. $a_n = \frac{(-1)^{n(n+1)/2}}{n^2}$

Find the rule:

9. $1, 4, 7, 10, \ldots$

11. $-1, 2, 7, 14, 23, \ldots$

13. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

In Exercises 25–46, determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit.

25. $a_n = \dfrac{n+1}{n}$

27. $a_n = (-1)^n\left(\dfrac{n}{n+1}\right)$

29. $a_n = \dfrac{3n^2 - n + 4}{2n^2 + 1}$

31. $a_n = \frac{n^2 - 1}{n + 1}$

33. $a_n = \frac{1 + (-1)^n}{n}$

35. $a_n = \cos \frac{n\pi}{2}$

37. $a_n = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$

39. $a_n = f^{(n-1)}(2),\ f(x) = \ln x$

n	1	2	3	4	5
$f^{(n-1)}(2)$	$\ln 2$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{2}{8}$	$-\frac{6}{16}$

$$a_n = \frac{(-1)^n (n-2)!}{2^{n-1}},\ n > 1$$

$n \geq 2$

41. $a_n = 3 - \dfrac{1}{2^n}$

definition of e
p. 360

43. $a_n = \left(1 + \dfrac{k}{n}\right)^n$

let $u = \dfrac{k}{n}$ $\quad n = \dfrac{k}{u}$

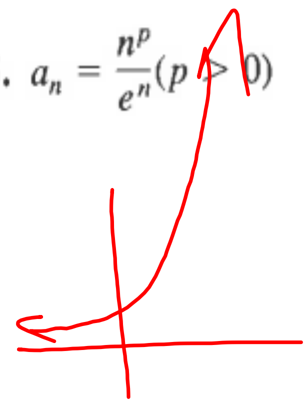
$$\lim_{n\to\infty}\left(1 + \frac{k}{n}\right)^n$$

$$\lim_{u\to 0}\left(1 + u\right)^{k/u}$$

$$\lim_{u\to 0}\left[\left(1 + u\right)^{1/u}\right]^k$$

$$e^k$$

45. $a_n = \dfrac{n^p}{e^n} \; (p > 0)$



Practice Test Answers:

1. C
2. D
3. C
4. B
5. C
6. B
7. A
8. D
9. C
10. D
11. B
12. D
13. B
14. D
15. E
16. E
17. D
18. D
19. A
20. C
21. C
22. A
23. B
24. D
25. A
26. C
27. E
28. E

Questions MC:

7) $2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$ $\quad u = \sqrt{x}$ $\quad du = \frac{1}{2\sqrt{x}}$

$= 2\int e^{u}\,du$

$2e^{\sqrt{x}} + C$

9) $s(t) = \int (4 - 6t^2)\,dt$

$s(t) = 4t - 2t^3 + C$

$7 = 4 - 2 + C$

16) $f'(x) = |x-2| \rightarrow$ all slopes are positive

eliminate A, B, C

slope @ $x = 2$ is 0

so eliminate D (gap)

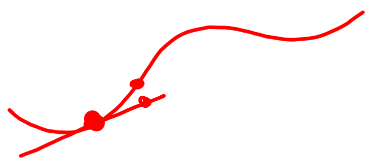
18) Tangent line @ $x = 1$

$m = f'(1) = 3$

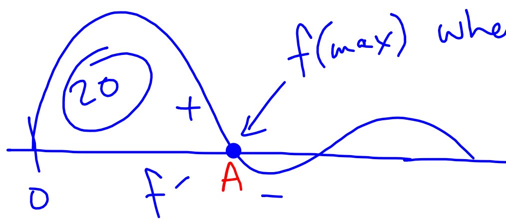
$y - 4 = 3(x - 1)$

$y = 3x + 1$

$f(1.2) \approx$



21)



f(max) where f' goes from + to −

$f(x) = \int f'(x)\,dx$

$f(0) = 2$

$f(max) = 2 + \int_0^A f'(x)\,dx$

$= 2 + 20$

23) A) $\lim_{x \to 0} (f(x) - 0) = 0$ T

B) $\lim_{x \to 0} \left(\frac{f(x)}{x}\right) = \frac{0}{0}$, L'Hôp

$\lim_{x \to 0} (f'(x))$

$\lim_{x \to 0^-} (f'(x)) = -1$

$\lim_{x \to 0^+} [f'(x)] = 1$

$\neq$

DNE

24) $\frac{dA}{dt} = 2\frac{dC}{dt}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$C = 2\pi r$

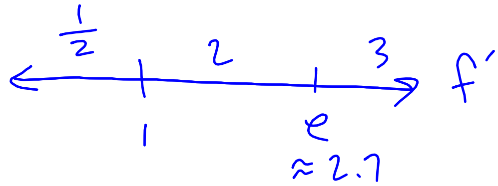
$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

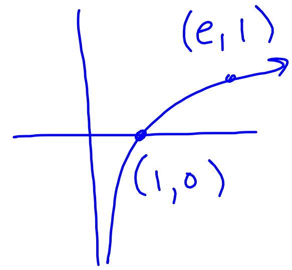
25) $x^2 y - 3x = y^3 - 3$

$2xy + x^2 \frac{dy}{dx} - 3 = 3y^2 \frac{dy}{dx}$

$\frac{2xy - 3}{3y^2 - x^2} = \frac{\frac{dy}{dx}\cancel{(3y^2 - x^2)}}{\cancel{3y^2 - x^2}}$

26)





27)

$$f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt \qquad \frac{d}{dx}[2x]$$

$$f'(x) = \sqrt{(2x)^2 - (2x)} \cdot 2$$

$$f'(2) = \left(\sqrt{16 - 4}\right)(2)$$

$$= 2\sqrt{12}$$

HW:

AP Practice - Multiple Choice with calculator.